Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE Mathematics

Pure 1 Paper 9MA0/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	Attempts $f(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3)=0$ leading to a correct equation in *a*.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3)=0$ method

$$\begin{array}{r l}
 & 3x^2+(2a-9)x+23-6a \\
x+3 \,\big) & 3x^3+2ax^2-4x+5a \\
 & \underline{3x^3+9x^2} \\
 & \quad (2a-9)x^2 -4x \\
 & \quad \underline{(2a-9)x^2 +(6a-27)x} \\
 & \qquad\qquad (23-6a)x+5a \\
 & \qquad\qquad (23-6a)x+69-18a
\end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in *x* are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

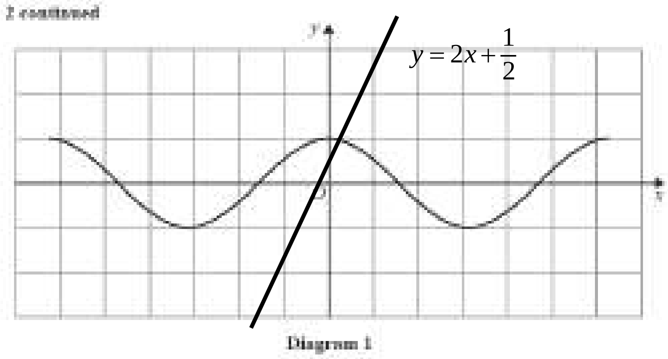
M1: This is scored for an attempt at solving a linear equation in *a*.
For the main scheme it is dependent upon having attempted $f(\pm3)=0$. Allow for a linear equation in *a* leading to $a=....$ Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{\big)3x^3+2ax^2-4x+5a}$ with quotient $3x^2...$ followed by a remainder in *a* set $=0\Rightarrow a=...$

or two terms in *a* are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Question	Scheme	Marks	AOs
2(a)		B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2 + \sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2 + \sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y=\dfrac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\dfrac{dy}{dx}=\dfrac{(x+1)^2\times(10x+10)-(5x^2+10x)\times2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\dfrac{dy}{dx}=\dfrac{(x+1)\times(10x+10)-(5x^2+10x)\times2}{(x+1)^3}=\dfrac{A}{(x+1)^3}$	M1	2.1
	$\dfrac{dy}{dx}=\dfrac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\dfrac{dy}{dx}=\dfrac{A}{(x+1)^n}, n=1,3$	B1ft	2.2a
		(1)	
			(5 marks)

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\dfrac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\dfrac{dy}{dx}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\dfrac{dy}{dx}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{\left((x+1)^2\right)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\dfrac{dy}{dx}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI $10x^3+30x^2+30x+10-10x^3-30x^2-20x$ but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

A1: $\frac{dy}{dx}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{dy}{dx}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{dy}{dx}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3\Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A,B\neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule $\Rightarrow\frac{dy}{dx}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	
		(5 marks)	

Question	Scheme	Marks	AOs

4 (a)	$\frac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \times (1 \pm ......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n = 1 + nax + \frac{n(n-1)}{2}a^2x^2 +$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert > 4$	B1	2.4
		(1)	
(b)(ii)	States $x = -\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1 \pm ax)^n = 1 \pm nax \pm \frac{n(n-1)}{2}a^2x^2 +$

Condone sign slips and the "*a*" not being squared in term 3. Condone $a = \pm 1$
Look for an attempt at the correct binomial coefficient for their *n*, being combined with the correct power of *ax*

A1: $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1 + \frac{x}{8} + \frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{1}{16}x + \frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2 + ..$

M1: For $4^{-\frac{1}{2}} + ...$ **M1:** As above but allow slips on the sign of *x* and the value of *n* **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.

Look for a suitable value and a suitable reason for both parts.

(b)(i)

B1: Requires $x=-14$ with a suitable reason.

Eg. $x=-14$ as the expansion is only valid for $|x|<4$ or equivalent.

Eg ‘ $x=-14$ as $|-14|>4$" or ' I cannot use $x=-14$ as $\left|\frac{-14}{4}\right|>1$ '

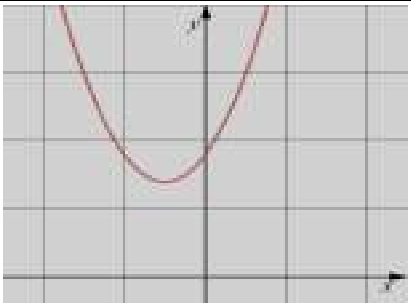
Eg. ‘ $x=-14$ as is outside the range $|x|<4$ ’

Do not allow ‘ -14 is too big’ or ‘ $x=-14, |x|<4$ ’ either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x=-\frac{1}{2}$ with a suitable reason.

Eg. $x=-\frac{1}{2}$ as it is ‘the smallest/smaller value’ or ‘ $x=-\frac{1}{2}$ as the value closest to zero’ (that will give the more accurate approximation). The bracketed statement is not required.

Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2$ & $b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $\mathrm{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\mathrm{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$ or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x)=f(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x)\to g(x)$ is $2x^2+4x+9\to 2(x-2)^2+4(x-2)+5$ So $g(x)=f(x-2)-4$
- $g(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x)=\dfrac{21}{\text{"}2(x+1)^2+7\text{"}}$ and attempts to find $\dfrac{21}{\text{their "7"}}$
- Attempts to differentiate, sets $4x+4=0\to x=-1$ and substitutes into $h(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=h(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<h(x)\leqslant 3$ Allow for $0<h\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<h(x)\leqslant\dfrac{21}{c}$

Question	Scheme	Marks	AOs
6 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\dfrac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\dfrac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \dfrac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\ 2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(**a**) **Option 1**

M1: For $V = Ae^{\pm kt}$ Do not allow if *k* is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 *V* and *t* must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20000$ into their exponential model

Candidates may start by simply writing $V = 20000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of *V* when *t* = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares *V* = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of *t* with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20000e^{-0.223t} \Rightarrow 2000 = 20000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to *k* then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 *V* and *t* must be used.

M1: Uses $t = 0 \Rightarrow A = 20\,000$ in their model. Alternatively uses $(0, 20000)$ and $(1,16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1,16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16\,000 = 20\,000 r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20\,000$

You may award if both number pairs $(0, 20000)$ or $(1,16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of *V* when *t* = 10. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares (£) 2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of *r* in the range $(0.8,1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8,1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20\,000 \Rightarrow A = 18\,000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16\,000 = 2\,000 + 18\,000e^k \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0.251t} + 2\,000$

(b)

M1: Uses their model to find the value of *V* when *t* = 10.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251 \times 10} + 2\,000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of *k* or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

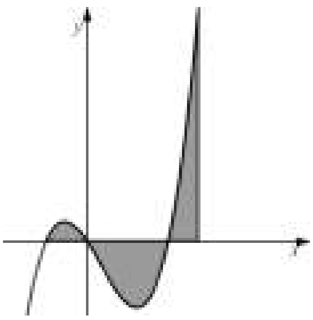
Condone 'make the value of *k* or the -0.251 greater (or less negative)'

....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k\,dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2\left(3b^2 - 20b + 20\right) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to $x^3 - 2x^2 - 8x$ (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2} = \frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1 = \frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients.

It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection $3b^4-8b^3-48b^2\pm80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out $3b^4-8b^3-48b^2\pm80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have = 0

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 (x = 5.442 may not be labelled.)

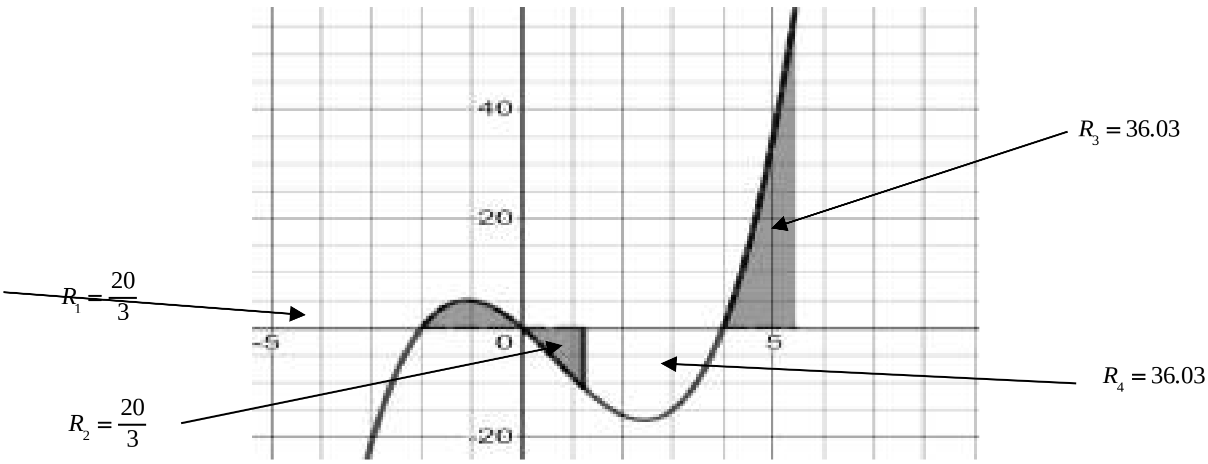
B1: Explains that (between $x=-2$ and $x=5.442$) the area above the x-axis = area below the x-axis with appropriate areas shaded or labelled**.**

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \rightarrow ...... \rightarrow ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \rightarrow a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(a)

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make '*a*' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in *a* are on the same side of the equation and the term in *b* is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \rightarrow \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$. They may state that *b* cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)

As a minimum accept that $b > 1$ as *a* cannot be negative.

Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

....

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

$$\log a - \log b = \log(a-b) \Rightarrow \log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b^2}{b-1} - b\right)$$

B1: Score for $\log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b}{b-1}\right)$

M1: Attempts to write $\frac{b^2}{b-1} - b$ as a single fraction $\frac{b^2}{b-1} - b = \frac{b^2 - b(b-1)}{b-1}$

Allow as two separate fractions with the same common denominator

A1*: Achieves lhs and rhs as $\log\left(\frac{b}{b-1}\right)$ **and** makes a comment such as "hence true"

Question 10

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 10 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up

(i**)**

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either *n* odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all *n*

Example of an algebraic proof

For $n=2m$, $n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1$, $n^2+2=(2m+1)^2+2=...$ FYI $\left(4m^2+4m+3\right)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2+m\right)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\dfrac{4m^2+2}{4} = m^2 + \dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\dfrac{n^2+2}{4} = \dfrac{4m^2+4m+3}{4} = m^2+m+\dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When *n* is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when *n* is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When *n* is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when *n* is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all $n.$	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction ‘Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ ’	M1	2.1
$\Rightarrow n^2 = 4k-2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then *n* is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all $n.$	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2} \times \sqrt{2k-1}$

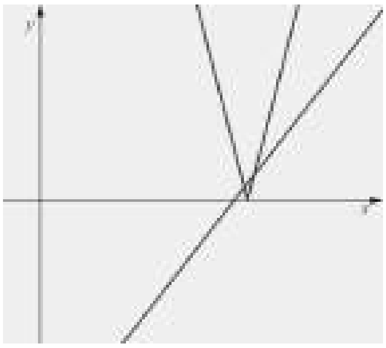
dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that *n* is irrational

Question 10 (ii)	Scheme	Marks	AOs

(ii)
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.
A1: and gives an example where it is true,
Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x = 9.4$ $\|3x-28\| = 0.2$ and $x - 9 = 0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x = 12$ $\|3x-28\| = 8$ $x - 9 = 3$ ✓	A1	2.4
	(2)	

Graphical Proof

 States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\|3x-28\| < x-9$ and points where $\|3x-28\| > x-9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x-28 < x-9$ and $-3x+28 < x-9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x-28 \geqslant x-9$ and $-3x+28 \geqslant x-9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^{\mathbf{1}}$ **6**th km is $6\times1.05\times1.05=6\times1.05^{\mathbf{2}}$ **7**th km is $6\times1.05\times1.05\times1.05=6\times1.05^{\mathbf{3}}$ Hence the time for the *r* th km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\frac{6.3(1.05^{16}-1)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\frac{6.3(1.05^{16}-1)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the *r* th km and the index of 1.05

Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\frac{a(r^n-1)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.

Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum 6\times1.05^n$ or $24+\frac{6(1.05^{20}-1)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on *a* and *n*

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum

The value of *r* must be 1.05 oe such as 105% but you should allow a slip on the value of *n* used for their value of *a* (See below: We are going to allow the correct value of *n* or one less)

If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a=6, \quad n=17$ or 16 Eg. $\frac{6(1.05^{17}-1)}{1.05-1}=(155.0)$ or $\frac{6(1.05^{16}-1)}{1.05-1}=(141.9)$

Allow for $a=6.3, n=16$ or 15 Eg $\frac{6.3(1.05^{16}-1)}{1.05-1}=(149.0)$ or $\frac{6.3(1.05^{15}-1)}{1.05-1}=(135.9)$

Allow for $a=6.615, \; n=15$ or 14 Eg $\frac{6.615(1.05^{15}-1)}{1.05-1}=(142.7)$ or $\frac{6.615(1.05^{14}-1)}{1.05-1}=(129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24+\frac{6.3(1.05^{16}-1)}{1.05-1}$, $18+\frac{6(1.05^{17}-1)}{1.05-1}$, $30.3+\frac{6.615(1.05^{15}-1)}{1.05-1}$

A1: For a total time of awrt 173 minutes and 3 seconds

This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

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Candidates that list values: Handy Table for Guidance

M1: For a correct overall strategy which would involve adding four sixes followed by at least 16 other values

The values which may be written in the form 6×1.05^2 or as numbers

Can be implied by $6+6+6+6+(6\times1.05)+.....+(6\times1.05^{16})$

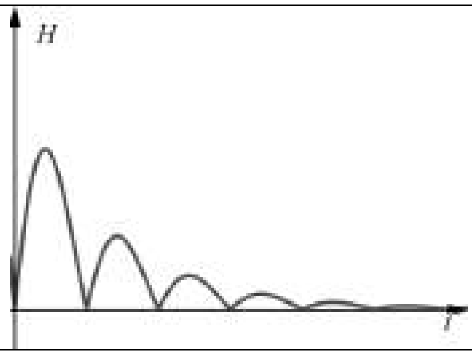
M1: For an attempt to add the numbers from (6×1.05) to (6×1.05^{16}). This could be done on a calculator in which case expect to see awrt 149

Alternatively, if written out, look for 16 values with 8 correct or follow through correct to 1 dp

A1: Awrt 173 minutes

A1: Awrt 173 minutes and 3 seconds

Km	Time per km	Total Time
1	6.0000	
2	6.0000	12
3	6.0000	18
4	6.0000	24
5	6.3000	30.3
6	6.6150	36.915
7	6.9458	43.86075
8	7.2930	51.15379
9	7.6577	58.81148
10	8.0406	66.85205
11	8.4426	75.29465
12	8.8647	84.15939
13	9.3080	93.46736
14	9.7734	103.2407
15	10.2620	113.5028
16	10.7751	124.2779
17	11.3139	135.5918
18	11.8796	147.4714
19	12.4736	159.945
20	13.0972	173.0422

Question	Scheme	Marks	AOs
12 (a)	$\text{f}(x) = 10\text{e}^{-0.25x}\sin x$		
	$\Rightarrow \text{f}'(x) = -2.5\text{e}^{-0.25x}\sin x + 10\text{e}^{-0.25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$\text{f}'(x) = 0 \Rightarrow -2.5\cancel{\text{e}^{-0.25x}}\sin x + 10\cancel{\text{e}^{-0.25x}}\cos x = 0$	M1	2.1
	$\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5} \Rightarrow \tan x = 4*$	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = \left\|10\text{e}^{-0.25\times 4.47}\sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .
So for example score expressions of the form $\pm ...\text{e}^{-0.25x}\sin x \pm ...\text{e}^{-0.25x}\cos x$ M1
Sight of $vdu - udv$ however is M0

A1: $\text{f}'(x) = -2.5\text{e}^{-0.25x}\sin x + 10\text{e}^{-0.25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $\text{f}'(x) = 0$, factorising/ cancelling out the $\text{e}^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$
Do not allow candidates to substitute $x = \arctan 4$ into $\text{f}'(x)$ to score this mark.

A1*: Shows the steps $\dfrac{\sin x}{\cos x} = \dfrac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4*$. $\dfrac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.
Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.
The intention should be that the graph should ‘sit’ on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt $76°$ or awrt $256°$ into $H(t)$

M1: Substitutes $t =$ awrt 4.47 into $H(t) = \left|10e^{-0.25t} \sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change''

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x-q=0$ when $x=2$ oe Hence $q=4$ *	B1*	2.4
	(ii) Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$*	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of *x*.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to *A* and *B*	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$ oe	A1	1.1b
	I = $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	I = $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}\,dx$ Or $[..............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8=3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why q = 4 and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of *x*.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to *A* and *B*

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct *A* and *B*

M1: Area *R* = $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)$

OR $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(x-2)+n\ln(x+3)$

Note that $\int \frac{l}{(x-2)}\,dx \to l\ln(kx-2k)$ and $\int \frac{m}{(x+3)}\,dx \to m\ln(nx+3n)$

A1ft: $= \int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their *A* and *B*

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}(dx)$ or $[.............]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once eg $\ln 6=\ln 2+\ln 3$, $\ln 8=3\ln 2$ or $m\ln 6k-m\ln 2k=m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3-4.8\ln 2$ oe

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

(a)

M1: Attempts to differentiate $x = 4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy} = k\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$ or $1 = k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x = 4\sin 2y \rightarrow y = ...\arcsin\left(\frac{x}{4}\right) \rightarrow \frac{dy}{dx} = \frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x = 8\sin y\cos y \Rightarrow \frac{dx}{dy} = \pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx} = \frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy} = 8\cos 2x \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x = 8y$ oe such as $x = 4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x = 8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m = \frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$, attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

....

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = \ldots \arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{dy}{dx} = \frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$

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